

AD-A085 644

RAND CORP SANTA MONICA CA
A PRELIMINARY ANALYSIS OF THE
MAR 80 G PARISH; W SOLLFREY
RAND/N-1260-AF

F/O 22/2
EFFECT OF WORK-AROUNDS ON SPACE S--ETC(U)
F49626-77-C-0023
ML

UNCLASSIFIED

1 OF 1

AD-A085 644



END
DATE
FILMED
7-80
DTIC

LEVEL II

12

A RAND NOTE

ADA 085644

A PRELIMINARY ANALYSIS OF THE EFFECT
OF WORK-AROUNDS ON SPACE SYSTEM PERFORMANCE
AND PROCUREMENT REQUIREMENTS--A PROPOSAL

Gary Parish, Lt. Col. USAF
William Sollfrey

March 1980

N-1260-AF

Prepared For

The United States Air Force

**DTIC
ELECTE
JUN 16 1980**



00 0 13 0 21

The research reported here was sponsored by the Directorate of Operational Requirements, Deputy Chief of Staff/Research, Development, and Acquisition, Hq USAF, under Contract F49620-77-C-0023. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

The RAND Corporation's Center for the Study of the Air Force and the Air Force Research and Development Community. The RAND Corporation is a nonprofit organization that provides research and analysis to the United States Government, the private sector, and the public. The RAND Corporation is a leader in the development of new ideas and solutions to complex problems. The RAND Corporation is a leader in the development of new ideas and solutions to complex problems.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER N-1260-AF	2. GOVT ACCESSION NO. AD-A085644	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Preliminary Analysis of the Effect of Work-Arounds on Space System Performance and Procurement Requirements--A Proposal		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lt. Col. G. Parish, USAF, W. Sollfrey		8. CONTRACT OR GRANT NUMBER(s) F49620-77-C-0023
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Rand Corporation 1700 Main Street Santa Monica, California 90401		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Requirements, Programs & Studies Group (AF/RDQM) Ofc, DCS/R&D & Acquisition Hq USAF, Washington, D. C. 20330		12. REPORT DATE March 1980
		13. NUMBER OF PAGES 27
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) No Restrictions		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Artificial Satellites Reliability Procurement Computer Systems Programs		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See Reverse Side		

DD FORM 1 JAN 73 1473

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

↙ This note identifies a factor that appears to have been neglected in explaining the discrepancy between predicted and observed satellite lifetimes in orbit. This factor--extension of satellite life through ingenious "work-around" corrections--is not represented in current satellite replenishment models. The note explores the effect of this omission on various measures of system performance, develops an analytical method of incorporating these work-arounds in replenishment models, and derives an initial estimate of the required parameters from a convenient data base. Incorporating the effect of work-arounds dramatically improves system performance and lowers procurement requirements. (JH)

↑

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

12

St. Louis, Mo.

12

五

Fig 31

(14)

The United States Air Force

15

DTIC ELECTE
JUN 16 1997

Rand
SANTA MONICA, CA. 90406

296637

PREFACE

This Note identifies a factor that appears to have been neglected in explaining the discrepancy between predicted satellite lifetimes and observed satellite lifetimes in orbit. This factor--extension of satellite life through ingenious, ad hoc "work-around" corrections--is described analytically, and an approach for including it in existing satellite replenishment models is discussed. The Note, which derives from work originally conducted under Project AIR FORCE's "Spacecraft Acquisition Strategies" study, was circulated in May 1978 as a working paper. Its reissuance in its present form--to reach a wider audience--has been delayed by the unavailability of the authors for the necessary rewriting and by the press of other work.

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DDC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or special
A	

PRECEDING PAGE BLANK-NOT FILMED

SUMMARY

The ingenious ways in which satellite designers, managers, and system controllers correct satellite malfunctions and anomalies which would otherwise result in satellite failure are not represented in current satellite replenishment models. In this Note the effect of this omission on various measures of system performance is explored, an analytical method of incorporating these "work-arounds" in current replenishment models is developed, and an initial estimate of the required parameters is derived from a convenient data base. It was found that incorporating the effect of work-arounds dramatically improves system performance and lowers procurement requirements. A more definitive study is suggested to provide the data needed by program managers to confidently include the effect of work-arounds in their replenishment studies.

PRECEDING PAGE BLANK-NOT FILMED

CONTENTS

PREFACE	111
SUMMARY	v
Section	
I. INTRODUCTION	1
II. REPRESENTATION OF SATELLITE LIFETIMES IN CURRENT REPLENISHMENT MODELS	2
III. THE EFFECT OF WORK-AROUNDS	4
IV. ESTIMATING THE INCIDENCE OF WORK-AROUNDS	8
V. SIMULATING THE EFFECT OF WORK-AROUNDS	10
VI. PROCUREMENT/PERFORMANCE IMPLICATIONS	13
VII. SOME QUALIFICATIONS AND A PROPOSAL	16
Appendix	
A. A SIMPLE METHOD OF DETERMINING SATELLITE REPLENISHMENT REQUIREMENTS	21
B. DERIVATION OF THE RELIABILITY FUNCTION WITH WORK-AROUNDS	23
REFERENCES	27

PRECEDING PAGE BLANK-NOT FILMED

I. INTRODUCTION

Within the satellite design, acquisition, and system management community the term "work-around" refers to the ingenious ways in which satellite designers, managers, and system controllers correct satellite malfunctions and anomalies which would otherwise result in satellite failure. The dedication and expertise of personnel at SAMSO, Aerospace Corporation, and the Air Force Satellite Control Facility and Test Center frequently enable satellites to function beyond their expected life through such work-arounds. However, this effect is nowhere included in the usual satellite replenishment models which are used to support satellite procurement requirements. The purpose of this Note is to describe the effect of work-arounds on effective satellite reliability and on system procurement requirements, and to suggest a way in which this effect can be incorporated into current replenishment models--specifically the Aerospace Corporation's Generalized Availability Model, GAP, and SAMSO(YAP)'s Life Cycle Cost and Orbital Spares Model. To accomplish this, it is necessary to inquire in more depth what the functions involved in work-arounds are for satellites in orbit; how such functions are executed; and the degree to which they are susceptible to some form of standardization so that, if desirable, the procedures can be made routine.

II. REPRESENTATION OF SATELLITE LIFETIMES IN CURRENT REPLENISHMENT MODELS

Current replenishment models generate representations of satellite lifetime by using random numbers in conjunction with various reliability functions. For instance, Rand's Satellite Availability Simulation Program uses two reliability functions--one for satellite lifetime as determined by the piece-part reliability function, and one for satellite lifetime as determined by wear-out phenomena--using the minimum of the two simulated lifetime numbers to represent satellite lifetime. Aerospace Corporation's Generalized Availability Program (Ref. 1) is similar, but adds a likelihood that infant failures will dominate the other two numbers, i.e.,

$$(1) \quad R = pR_i + (1 - p)R_p R_w$$

where p = probability of infant failure,
 $R_p(t)$ = piece-part reliability function,
 $R_w(t)$ = wear-out reliability function,
 R_i = infant reliability function.

Often, R_p will consist of the product of reliability functions for various payloads, and GAP includes the ability to generate satellite life assuming a criterion that m out of n payloads survive.

For the purposes of this paper, the simplest case will be assumed in which there is a single payload whose lifetime is entirely characterized by the piece-part reliability function, $R_p(t)$. The operation of current replenishment models is then illustrated in Fig. 1, which shows the piece-part reliability function.

As shown in Fig. 1, conventional models generate a random number which is uniformly distributed on the interval $[0, 1.0]$. This number is transformed through the inverse of the reliability function into a single number representing the age of the satellite at the time of failure.

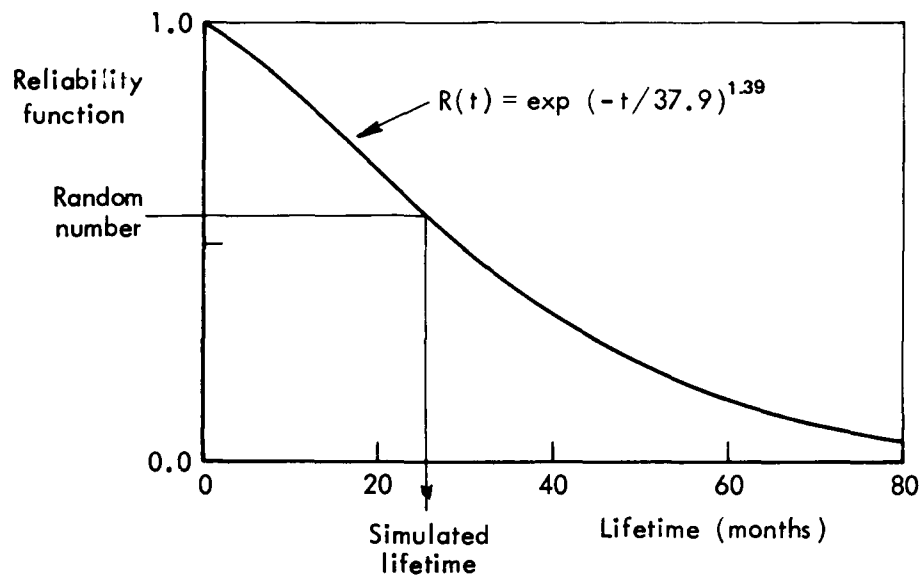


Fig. 1—Satellite lifetime generation in current replenishment models

III. THE EFFECT OF WORK-AROUNDS

Work-arounds have the effect of permitting a satellite to "fail" more than once and to continue operating until a failure occurs which cannot be worked around. The effect is shown in Fig. 2 for the case of two successful work-arounds. In general, to represent this situation the reliability function of the satellite, $R_k(t)$, given k work-arounds, must be derived from the basic satellite reliability function, $R(t)$. Letting $P_f(t) = 1 - R(t)$, it can be shown that

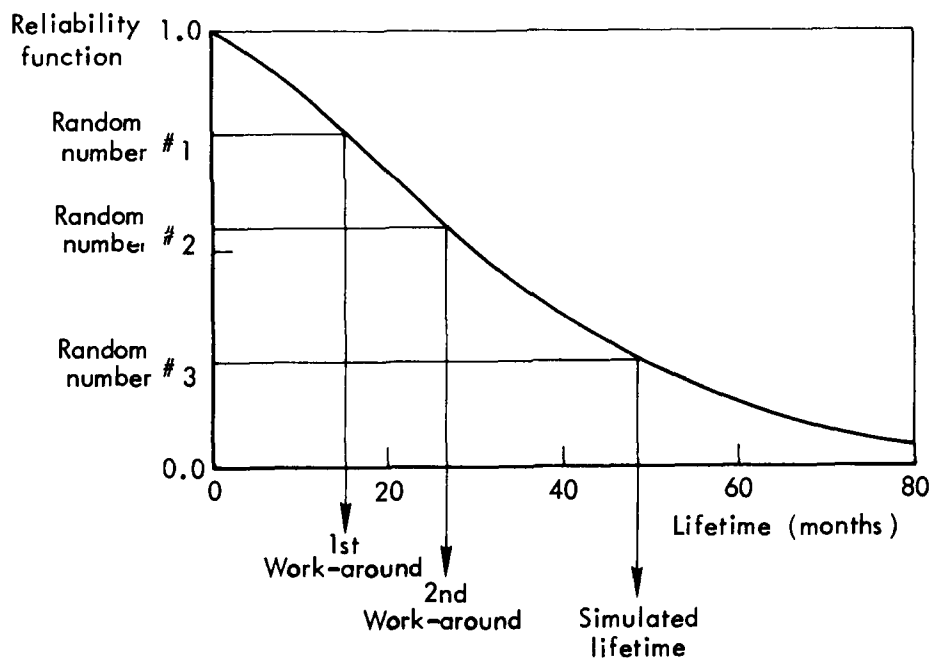


Fig. 2—Satellite lifetime generation with work-around effects
(two work-arounds)

$$R_k(t) = 1 - \int_0^t d\tau_1 \int_{\tau_1}^t d\tau_2 \cdots \int_{\tau_{k-1}}^t P_f(t)_{\tau_1, \dots, \tau_k} d\tau_k$$

where

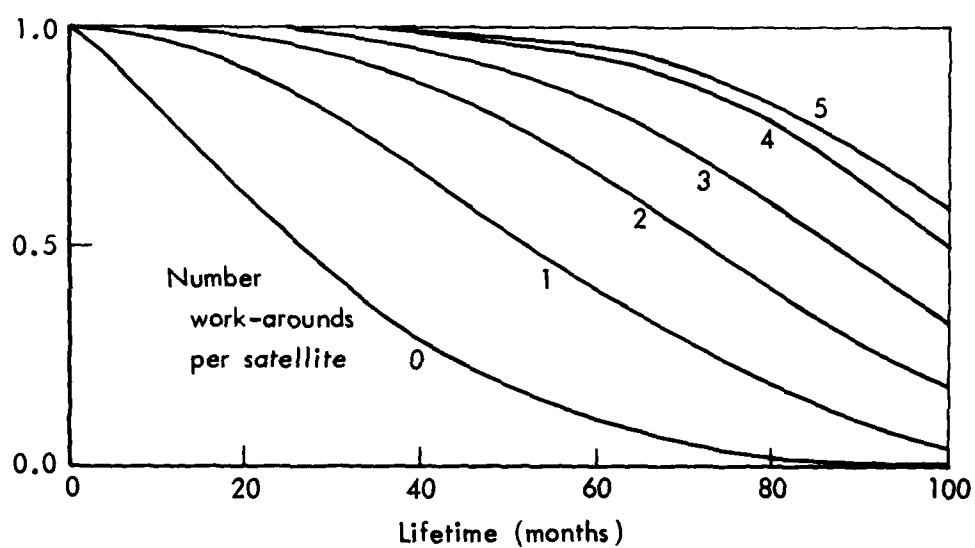
$$P_{f_{\tau_1, \dots, \tau_k}}(t) = P_f(0, \tau_1) P_{f_1}(\tau_1, \tau_2) \cdots P_{f_k}(\tau_k, t)$$

and $P_{f_i}(\tau_i, \tau_{i+1})$ = the probability of failure in the interval (τ_i, τ_{i+1}) given that a work-around occurred at τ_i . This is $P(t)$ conditional on the occurrence of i work-arounds.

It is shown in Appendix B that when the occurrence of work-arounds does not influence the reliability function, $R(t)$, $R_k(t)$ is given by the expression

$$R_k(t) = R(t) \sum_{n=0}^k \frac{[-\ln R(t)]^n}{n!}.$$

This is a reasonable assumption if the satellite contains many parts and the work-arounds compensate for only those parts that have failed. In the calculations which follow, it is assumed that $P_{f_i} = P_f$, i.e., the reliability function is such that the work-arounds do not influence the reliability of the satellite. With this assumption, the above integral can be easily approximated using computer simulation techniques, i.e., simply by multiplying together $N_w + 1$ random numbers with uniform distribution on the interval $[0, 1]$ and using the result to determine a satellite lifetime through the inverse of the reliability function, $R(t)$. Figure 3 shows the results of using this technique to calculate the effective life of satellites used in a typical program for various numbers of assumed work-arounds. The effect of incorporating work-arounds dramatically changes the effective reliability of the satellite. For this real-world example, one work-around effectively increases satellite mean-life by a factor of 1.7; two work-arounds increase it by 2.3, as shown in Table 1. As a gross rule of thumb, procurement requirements vary as the inverse of the ratio provided in this table. The procurement requirements shown in Table 1 are based on the simple rule of thumb shown in



Piece-part reliability $R(t) = \exp(-t/39.7)^{1.39}$

Wear-out reliability $R(t) = \exp(-t/125)^{6.0}$

Fig. 3—Effective satellite lifetime distributions
for various numbers of work-arounds

Table 1
EFFECTIVE MEAN SATELLITE LIFE AS A FUNCTION
OF THE NUMBER OF WORK-AROUNDS^a

Number of Work-arounds	Mean Satellite Life (mo.)	Ratio ^b	Satellites for 4 Satellite Systems ^c
0	34.6	1/1	14
1	59.5	1.7/1	8
2	80.9	2.3/1	6
3	100.3	2.9/1	5
4	118.3	3.4/1	4
5	135.3	3.9/1	4

^aThis table provides the mean of the work-around distribution of the reliability function shown in Fig. 1.

^bThe ratio of mean satellite life with work-arounds to mean satellite life with no work-arounds.

^cFor 10 years of operation, assuming perfect satellite launch/initialization probability, and all satellites have the same number of work-arounds.

Equation (2) and do not include considerations of system performance such as system availability, expected maximum system outage, etc.

$$(2) \quad N = \frac{T \cdot N_{sta}}{P_L \cdot MMD}$$

In Eq. (2),

N = the number of satellites required,
T = the duration of the program,
N_{sta} = the number of orbital stations
that must be operational to
meet mission requirements,
P_L = the satellite launch/initial-
ization probability,
MMD = the satellite mean-mission duration.

Thus, when work-around effects are considered, the effective mean-mission duration depends on the number of work-arounds expected per satellite. This modifies the rule of thumb provided in Eq. (2). A version of Eq. (2) which incorporates the effect of work-arounds is provided in Appendix A. Also, it is important to note that *comparisons of achieved satellite life vs. predicted satellite life which fail to consider the effect of work-arounds may lead to spurious conclusions about the efficacy of satellite reliability functions for predicting satellite lifetimes.*

IV. ESTIMATING THE INCIDENCE OF WORK-AROUNDS

In this section we estimate the frequency of occurrence of work-arounds assuming that work-arounds are generated by a Poisson process, i.e., for a given satellite the number of work-arounds experienced, n , is governed by

$$P(n) = \frac{(\lambda)^n e^{-\lambda}}{n!} \quad n = 0, \dots, N_{\max}, \quad \lambda > 0.$$

A high confidence estimate of the work-around distribution, $P(n)$, $n = 0, \dots, N_{\max}$ should be derived from a comprehensive review of U.S. satellite experience, and is recommended in Section VII. Lacking this, we derive a preliminary estimate of λ for the Poisson distribution using a convenient data base consisting of ten programs which include 25 satellites for a total of 50.9 orbit years, as shown in Table 2.* Of the 35 major anomalies shown, 18 meet the criteria for work-arounds listed in Section VII below. For this sample, then, there are 18 work-arounds for 35 satellite orbit years or for 25 satellites. The Poisson parameter is therefore estimated to be:

$$\begin{aligned} \hat{\lambda} &= 18/(611/12) = 0.35 \text{ work-arounds per orbit year, or} \\ \hat{\lambda} &= 18/25 = .72 \text{ work-arounds per satellite.} \end{aligned}$$

Judging from Table 1, this value of λ can be expected to significantly affect satellite procurement requirements. A more complete and accurate discussion of this effect is provided in Section VI.

* *Standardization and Program Practices Analysis--Final Report*, Aerospace Corporation Report No. ATR-77(7353-01)-1, Vol. II, March 1977.

Table 2
PROGRAM DATA FOR EVALUATION OF PROGRAM SUCCESS

Program: Flight Number:	DSCS-II				DSP, Phase I				STP (\$3)			STP P72-1	SMS			ITOS		OSO	Number of		ATS	Pioneer		STP P71-2
	1	2	3	4	1	2	3	4	1	2	3	1	1	2	3	1	4	8	5	6	6	10	11	1
Anomalies Major Weighted Anom. Failure Report Period (Months) Mission Life (Months)																								
	5	7	1	1	5	2	0	1	1	0	1	0	2	0	1	1	2	1	0	0	2	0	0	2
	1.43				0.37				0.65			0	0.58			1.20		0.83	0		0.95	0		0.74
	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	
	(1)	(1)	(1)		(1)				6.8	18.0	6.0	14	(5)			(1)	(1)	12	51.6	21.0	21	34.5	26.3	(1)
	19.0	10.1	32.4	36.6	40.1	69.9	60.8	45.5	6				19.7	19.8	12.0	16.8	8.3	12						25.1
60				36								12	60			12	12	12	12	60	60		6	
Spacecraft Delivered																								
Actual	4				4				3			1	3			8		1	2		1.6	?		1
Planned	4				3				3			1	2			4		4	2		2	2		3
Schedule (Month) (2)																								
Actual	33				55				21			18	60			33		49	32		45	27		16.7
Planned	23				42				17			15	30			25		32	32		31	27		17.4
Cost (\$M)																								
Actual	84.6				85				11.1			6.6	44.8			77.3		36.0	59.0		112.1	50.4		(4)
Planned	56.8				46				5.3			5.2	12.4			27.5		21.7	42.2		55.5	28.1		(4)

- (1) Time to flight failure or termination.
 (2) Time to first launch.
 (3) Includes a prototype. This was changed to only a prototype.
 (4) Includes Agena stage.
 (5) Spacecraft placed in standby on 9 January 1976 and reported to be satisfactory 4 months later in standby mode.

SOURCE: "Standardization and Program Practice Analysis-Final Report,"
 Aerospace Corporation Report No. ATR-77(7253-01)-1, Vol. II,
 March 1977.

V. SIMULATING THE EFFECT OF WORK-AROUNDS

Incorporating the effect of work-arounds into the lifetime number generator in current satellite replenishment models appears to be quite easy. Each time such a number is required in the simulation, a two step procedure is followed:

- Step 1. Simulate the number of work-arounds, N_w , the satellite will experience.
- Step 2. Generate $N_w + 1$ random numbers from a uniform $[0,1]$ distribution, form the product, and use the result to generate the lifetime number of the satellite.

This approach requires that a probability distribution, $P(n)$, for the number, n , of work-arounds be input into the replenishment model. Given $P(n)$, this procedure is easy to implement into existing satellite replenishment models.

A Poisson distribution could be assumed for modeling Step 1 above; i.e., the probability of any given number, n , of work-arounds for any satellite is given by:

$$(3) \quad P(n) = \frac{\lambda^n e^{-\lambda}}{n!}, \quad n = 0, 1, \dots, \infty$$

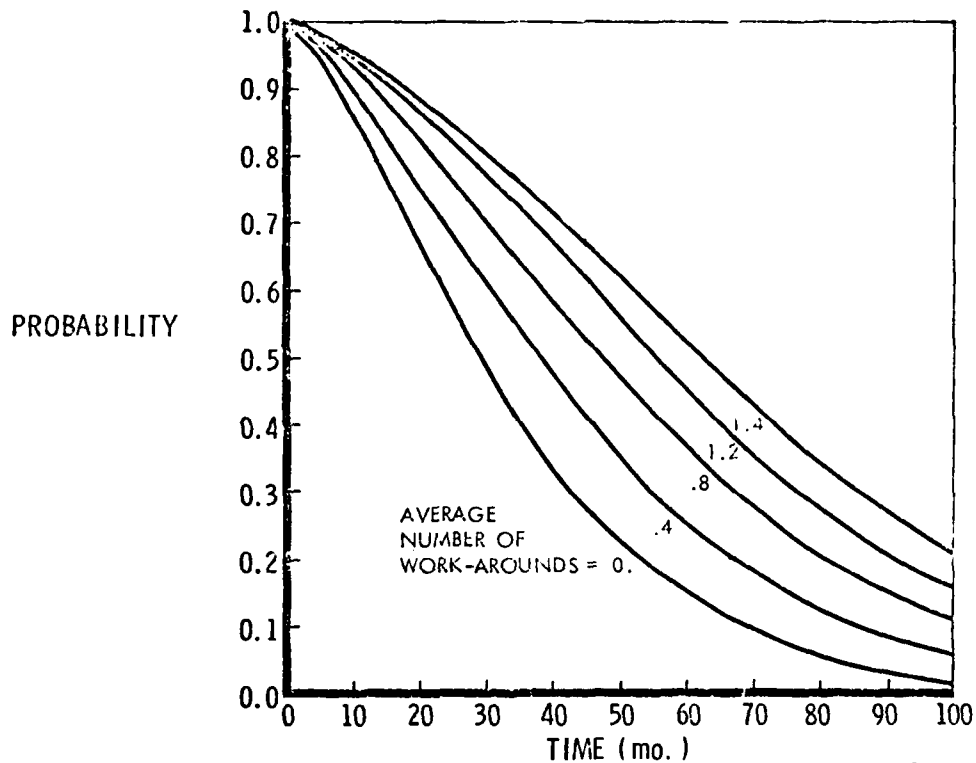
where λ is the average number of work-arounds per satellite. This is a simplifying assumption which requires only the estimation of the parameter λ rather than the estimation of $P(n)$ for every value of n in the absence of such an assumption. One would expect to find some deviation from the Poisson due to the upper limit on the number of work-arounds per satellite which would inflate the value of $P(0)$ and reduce the value of $P(n)$ to zero for values of n greater than this upper limit. For values of λ derived later in this paper, this deviation is insignificant.

The effective reliability function given an arbitrary distribution of work-arounds is given by:

$$(4) \quad R_e(t) = R(t) \sum_{n=0}^{N_{\max}} p(n) \sum_{m=0}^n \frac{[-\ln R(t)]^m}{m!},$$

which is the weighted average of $R_n(t)$ where the weights are merely $p(n)$. The effective reliability function for the satellite used in Fig. 2 is shown in Fig. 4.

EFFECTIVE SATELLITE RELIABILITY AS A FUNCTION OF AVERAGE NUMBER OF WORK-AROUNDS



Satellite Reliability Function, $R(t) = \exp(-t/37.5)^{1.39}$
Maximum Number of Work-arounds/satellite = 5

Fig. 4

VI. PROCUREMENT/PERFORMANCE IMPLICATIONS

Table 1 provides a rough indication of the effect of work-arounds on the size of the procurement buy for a typical four-satellite system with ten-year program life. A more precise estimate of the effect on system procurement and system performance requires the use of computer replenishment models. Rand's Satellite Availability Simulation Model has been modified to incorporate the changes suggested in Section IV. The effect of incorporating work-arounds depends on the value of the Poisson parameter, λ . Figure 5 shows the effect of work-arounds on various measures of system performance with λ as a parameter for a typical satellite program. As expected, system performance in every category is improved when the effect of work-arounds is included. The expected improvement for a nominal 13-satellite system is shown in Table 3, as a function of the average number of work-arounds. The conclusion to be drawn from Table 3 and Fig. 5 is that Air Force

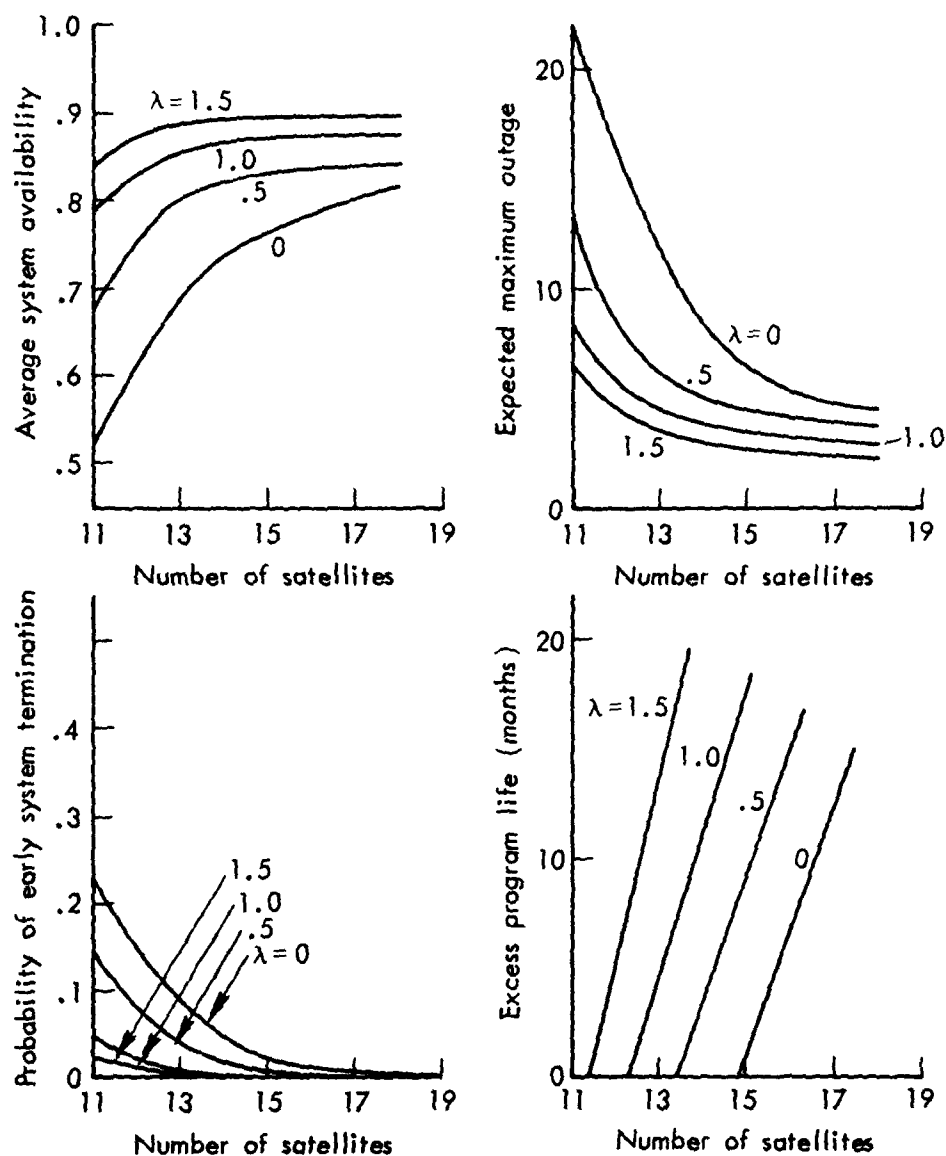
Table 3

SYSTEM PERFORMANCE FOR A 13-SATELLITE SYSTEM^a

Performance Parameter	Average Number of Work-Arounds			
	$\lambda =$			
	0	.5	1.0	1.5
Average Availability (%)	68.5	80.5	85.0	88.5
Expected Maximum Outage (mo)	11.7	6.0	4.4	3.5
Probability of Early Termination (%)	9.5	4.0	1.2	.7
Excess Program Life (mo)	-10.0	-2.0	5.0	14.0

^aFour satellite system for a 10-year program.

requirements based on GAP-type analyses which fail to include the effect of work-arounds may seriously understate expected system performance. For example, to achieve an 80 percent system availability current methods of determining requirements would suggest that 17 satellites are required ($\lambda = 0$ in Fig. 5). However, when the effect of work-arounds is included ($\hat{\lambda} = 0.7$ in Fig. 5) only 12 satellites



Number of stations = 4
 Station start times = 0, 2, 4, 6 mo.
 Booster reliability = .87
 Launch response time = 2 mo.
 S/C initialization = 0.94

Piece-part reliability
 $R(t) = \exp(-t/37.9)^{1.39}$
 Wear-out reliability
 $R(t) = \exp(-t/62.4)^{6.0}$
 Number of iterations = 400

Fig. 5—The effect of the average number of work-arounds (λ) on system performance

are required. This represents a savings of 30 percent in the number of satellites required, or a dollar savings of roughly \$175 million for this 10-year program. Since GAP-type analyses--including the Integrated Life Cycle Cost Model--omit the effects of work-arounds, similar savings on most of the other satellites being procured by the Air Force could be realized.

It is important to note that the occurrence of work-arounds is independent of the satellite reliability function, although the significance of the number of work-arounds depends directly on the reliability function. Thus, correcting the procurement/decision process for the failure to consider work-around effects can proceed independent of efforts to improve the accuracy of the reliability function as a predictor of satellite life.

VII. SOME QUALIFICATIONS AND A PROPOSAL

The importance of work-arounds to satellite procurement decisions and satellite system performance is a function of the value of the Poisson parameter, λ . For λ very close to zero, the effect is negligible; however, for values as small as $\lambda = 0.2$ the effect becomes significant. The estimate derived herein is only a very rough estimate, but it indicates that a more rigorous estimating effort should be pursued because the potential payoff to the Air Force may be quite high. Working-level SAMSO and Aerospace personnel concur in this assessment and indicate that such a study has not been performed. A study to provide such an estimate seems advisable. Such a study would address the following questions:

- (1) What precisely is a "work-around"? Obviously it refers to activities and procedures, the effects of which are not captured by the reliability function, but how can the concept be operationalized for survey and data collection purposes? In the proposed study the following definition would be used:

Work-around--a satellite malfunction, anomaly, or part failure that would result in satellite failure or severe degradation of satellite mission accomplishment except for the ingenious activities of satellite designers, system managers, and system controllers in compensating for the problem.

- Excludes parts failures, malfunctions, and anomalies which can be corrected by switch-over to redundant units or back-up modes of operation which are captured in the satellite reliability function.
- Excludes parts failures, malfunctions, and anomalies associated with infant mortalities, i.e., satellite failures during the first six months of operation.

- Includes procedural and software changes not captured in the satellite reliability function.
- Includes malfunctions, anomalies, and parts failures which have minor effect on satellite mission accomplishment but which may significantly affect the wear-out characteristics of the system. (Because satellite wear-out is rare, this type of wear-out is less important than the other types of wear-outs. This type will be denoted as Category II wear-outs and the other, more important type, will be denoted as Category I wear-outs. It is important to make this distinction because of the way the two types will be used in simulation models.)

- (2) What is the relationship between the number of work-arounds and satellite lifetime? In this paper the implicit assumption is that the more work-arounds per satellite, the longer the satellite lifetime. Are there sufficient data to reject this hypothesis? The data in Table 2 show that 14 out of 35 serious orbital anomalies occurred on four satellites in one program and the lifetimes of these satellites were significantly less than the design life. The question of what program/satellite factors significantly influence the incidence of orbital problems requiring work-arounds should also be addressed. An investigation of the allocation of the scarce resources of the Technical Assistance Groups at Aerospace Corporation, which perform the work-around function, would be appropriate. Perhaps this important function is being underfunded, or distorted by dysfunctional organizational incentives.

- (3) What is the actual work-around distribution, $P(n)$?
Is a Poisson approximation to $P(n)$ adequate? If so, what value of λ should be used in specifying satellite requirements? Should λ be uniform across programs or program specific? If program specific, what are the factors which determine the appropriate value of λ for a given program? It would seem that programs which enjoy higher priorities and can command higher levels of support from the Technical Assistance Groups and the Satellite Control Facility should use higher values of λ in their replenishment calculations than other programs less favorably endowed.
- (4) Is it possible in satellite system design to enhance the probability of successful work-arounds?
- (5) Is it possible that by incorporating the work-around concept into existing satellite replenishment models one can derive a quantitative estimate of the intrinsic dollar value of the functions of the Technical Assistance Groups and the Satellite Control Facility?
- (6) Assuming that the work-around distribution is a viable concept which should be incorporated into the determination of Air Force satellite requirements, how should it be done? What implementation steps should be considered? What institutional and organizational factors should be addressed?

Work-arounds are not without costs. Since they involve software and procedural changes, they affect support requirements placed by the SPO on the Satellite Control Facility. The capacity of the Satellite Control Facility to service the work-around requirements of a growing satellite population in the next decade may provide the ultimate limit on the work-around distribution, $P(n)$.

A study which could answer the above questions could be performed at a one-man-year level of effort and be completed in one year. The

approach would be to collect all orbital satellite failure/anomaly reports for a broad spectrum of satellite programs. The failure/anomaly reports would be screened to select those which meet the work-around definition. Time of work-around occurrence and time to loss of function or satellite operation would be noted. Work-arounds would also be categorized with the help of Aerospace, contractor, and SPO personnel according to mission impact. The work-around distribution, $P(n)$, would be estimated from the resulting data, and the impact of incorporating work-arounds into the replenishment simulations for representative satellite programs would be estimated using existing simulation models. The cooperation of the Air Staff, SAMSO/Aerospace, and Air Force Satellite Control Facility/Satellite Test Center would be essential. Documentation would require an additional six months.

Appendix A

A SIMPLE METHOD OF DETERMINING
SATELLITE REPLENISHMENT REQUIREMENTS

This method assumes that the work-around distribution is Poisson with parameter λ ; i.e.,

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!} \quad n = 0, 1, \dots, N_{\max}.$$

It is also assumed that the satellite reliability function can be expressed in the form of a Weibull function, $R(t) = \exp(-t/\alpha)^\beta$. The mean life of the satellite given k work-arounds is:

$$\overline{L(k)} = (\alpha/\beta) \sum_{n=0}^k \frac{\Gamma(n+\beta-1)}{n!},$$

as shown in Appendix B.

Then the expected number of satellites required is given by

$$(5) \quad n = \frac{TS}{P_L} \left[(1+\epsilon) \frac{P(0)}{L(0)} + \sum_{n=1}^{N_{\max}} \frac{p(n)}{L(n)} \right]$$

where

N = the number of satellites required,

S = the number of orbital stations required for the space mission,

T = the duration of the program,

P_L = the probability of satellite launch/initialization,

ϵ = a correction factor which compensates for the finite number of work-arounds a given satellite can experience, i.e.,

$$\epsilon = \left[1 - \sum_{n=0}^{N_{\max}} p(n) \right],$$

λ = the average number of work-arounds per satellite,

α, β = the parameters of the Weibull approximation to the satellite reliability function.

PRECEDING PAGE BLANK-NOT FILMED

This method is suitable for implementation on programmable hand calculators. It is interesting to note that this method provides the same results as Eq. (2) in Section III provided that the MMD used in that equation is the MMD appropriate for the average number of work-arounds, λ . Unfortunately Eq. (2) is incapable of handling situations in which λ is not an integer, or for large (>3) values of λ .

Appendix B

DERIVATION OF THE RELIABILITY FUNCTION WITH WORK-AROUNDS

The method of derivation is demonstrated sufficiently for the case of two work-arounds depicted in Fig. 2. First, choose a random number u_1 uniformly distributed on the range $[0, 1]$. The probability density of u_1 is just du_1 . Second, choose a random number u_2 uniformly distributed on the range $[0, u_1]$ corresponding to the second random number in Fig. 2. The probability density of u_2 is du_2/u_1 , since it must be normalized to unity over the range $[0, u_1]$. Then choose a random number u_3 uniformly distributed over the range $[0, u_2]$, whose probability density will be du_3/u_2 . The simulated lifetime of the system will be $t(u_3)$, where $t(R)$ is the inverse function of $R(t)$, as shown in the figure. The expected value of the lifetime, T_2 , denoting two work-arounds, is found by averaging over the permissible values of the three random numbers, and is:

$$T_2 = \int_0^1 du_1 \int_0^{u_1} \frac{du_2}{u_1} \int_0^{u_2} \frac{du_3}{u_2} t(u_3) .$$

Integrate by parts on u_1 , and we have:

$$T_2 = (\ln u_1) \int_0^{u_1} \frac{du_2}{u_2} \int_0^{u_2} du_3 t(u_3) \Big|_{u_1=0}^{u_1=1} - \int_0^1 du_1 \frac{\ln u_1}{u_1} \int_0^{u_1} du_3 t(u_3) .$$

The integrated term vanishes at $u_1 = 1$ because $\ln 1 = 0$, and at $u_1 = 0$ because the integral vanishes faster than $\ln u_1$ increases. Again integrate by parts, and we have:

$$T_2 = -\frac{1}{2} (\ln u_1)^2 \int_0^{u_1} du_3 t(u_3) \Big|_{u_1=0}^{u_1=1} + \frac{1}{2} \int_0^1 du_1 (\ln u_1)^2 t(u_1) .$$

As before, the integrated term vanishes. It is clear that the same process may be applied if there are k work-arounds, with the result:

$$T_k = \frac{(-1)^k}{k!} \int_0^1 du_1 (\ln u_1)^k t(u_1) .$$

Since $t(u_1)$ is the inverse function to $R(t)$, the time may be introduced as the independent variable in the integral. The limits $(0, 1)$ for u_1 become $(\infty, 0)$ for t . There results:

$$T_k = -\frac{1}{k!} \int_0^\infty dt t (-\ln R(t))^k dR/dt .$$

We also have, from the definition of the reliability function $R_k(t)$:

$$T_k = -\int_0^\infty dt t dR_k/dt ,$$

because the probability the system with k work-arounds survives to t and then fails is $-dR_k/dt$. Equating the two expressions yields:

$$\frac{dR_k}{dt} = \frac{(-\ln R(t))^k}{k!} \frac{dR}{dt}$$

$$R_k(t) = 1 + \int_0^t dt' \frac{(-\ln R(t'))^k}{k!} \frac{dR(t')}{dt'} .$$

Still another integration by parts yields:

$$R_k(t) = 1 + \frac{(-1)^k}{k!} \left[R(t') (\ln R(t'))^k \right]_{t'=0}^{t'=t}$$

$$= 1 + \frac{(-1)^k}{k!} \left[\int_0^t dt' R(t') \cdot k \cdot \frac{(\ln R(t'))^{k-1}}{R(t')} \frac{dR(t')}{dt'} \right]$$

$$= 1 + \frac{R(t) (-\ln R(t))^k}{k!} + \int_0^t dt' \frac{(-\ln R(t'))^{k-1}}{(k-1)!} \frac{dR(t')}{dt'}$$

$$= R_{k-1}(t) + R(t) (-\ln R(t))^k / k! .$$

This recursion relation may be summed, starting with $R_0(t) = R(t)$, to yield the result in the text:

$$R_k(t) = R(t) \sum_{n=0}^k \frac{[-\ln R(t)]^n}{n!}.$$

When the original reliability function is a Weibull distribution, the mean lifetime T_k can be found as a simple expression. With

$$R(t) = \exp \left[- (t/\alpha)^\beta \right],$$

the integrations for the terms in the series representation of $R_k(t)$ yield gamma functions, with the result:

$$T_k = \frac{\alpha}{\beta} \sum_{n=0}^k \frac{\Gamma\left(n + \frac{1}{\beta}\right)}{n!}.$$

For the constants appropriate to the illustrative program described on page 13 [$\alpha = 37.9$, $\beta = 1.39$], we have determined the mean lifetime from the formula above. Plotting the curves for $R_k(t)$ and reading the time when $R_k(t) = 0.5$ determines the median lifetime. The results for up to three work-arounds are listed in Table B.1.

Table B.1
MEAN AND MEDIAN LIFETIMES

Number of Work-Arounds	Mean Lifetime	Median Lifetime
0	34.6	29.0
1	59.5	55.2
2	80.9	77.2
3	100.2	96.5

These results may be compared to the Monte Carlo simulations in the text.

REFERENCE

1. Sugihara, S., and H. Wong, *System Availability Analysis*, The Aerospace Corporation, Report No. TOR-0066(5129-01)-17, March 20, 1970.

PRECEDING PAGE BLANK-NOT FILMED